

A New Optimal Control System Design for Chemical Processes^{*}

CONG Erding (丛二丁)^{1,2}, HU Minghui (胡明慧)^{1,**}, TU Shandong (涂善东)¹ and SHAO Huihe (邵惠鹤)³

¹ Key Laboratory of Pressure Systems and Safety of Ministry of Education, East China University of Science and Technology, Shanghai 200237, China

² Petro-China Jilin Petrochemical Company, Jilin 132021, China

³ School of Electronic, Information and Electrical Engineering, Shanghai Jiaotong University, Shanghai 200240, China

Abstract Based on frequency response and convex optimization, a novel optimal control system was developed for chemical processes. The feedforward control is designed to improve the tracking performance of closed loop chemical systems. The parametric model is not required because the system directly utilizes the frequency response of the loop transfer function, which can be measured accurately. In particular, the extremal values of magnitude and phase can be solved according to constrained quadratic programming optimizer and convex optimization. Simulation examples show the effectiveness of the method. The design method is simple and easily adopted in chemical industry.

Keywords control system, frequency response, convex optimization, chemical processes

1 INTRODUCTION

In recent years, optimal control systems have been strongly demanded in chemical processes [1–3]. In the control system design, models from response function measurements form the basis for structural dynamic analysis of complex system [4–6]. The model could be used for prediction, feedback controller design and feedforward controller design such as the zero phase error tracking controller [7, 8] and optimal feedforward control [9–12]. In practice, such precise information may be unavailable or is difficult to obtain, while frequency response of a plant can easily be measured. On the other hand, it is an essential part of controller design in classical control design such as that for proportion-integral-derivative (PID) controllers and their tuning rules [13, 14].

It is unlikely that the model matches the measured frequency response, and some modeling error may affect parametric fitting. For feedforward controller design, this error can be avoided by using the frequency response measurement directly [15, 16]. The objective of this work is to find a controller so that the system behavior from the reference signal to the output signal presents a desired frequency response. The extremal values of the magnitude and phase of the family frequency response, which is generally involve in convex optimization problem, can be solved by a constrained quadratic programming optimizer [17].

In this paper, we present an efficient and practical method for design. We consider the transfer function applying convex optimization to the frequency response data in a time domain filter, which can be convolved with an input signal to generate a prediction of output. For feedforward control, the system frequency response can be inverted on a point-by-point basis. The

feedforward controller complete set can be obtained in terms of frequency response data of the closed loop system rather than from transfer function or state space of the loop model.

2 MODEL DEVELOPMENT

2.1 Statement of problem

Figure 1 shows the control system for a chemical process, where r_F is the input, $P(s)$ represents the dynamics in chemical process of a single input single output (SISO) system, $C(s)$ is an existing feedback controller, $F(s)$ is the feedforward controller to be designed, $W_e(s)$, $W_u(s)$ and $W_T(s)$ are frequency-varying weighting functions for control error e , control effort u and process output y , respectively.

The existing feedback system (within the dashed lines) has one input r_F , with the control system transfer function as follows

$$G(s) = C(s)P(s) = \frac{N(s)}{D(s)} \quad (1)$$

The loop input-output napping can be expressed by a transfer function matrix

$$\begin{aligned} \mathbf{G}_{cl}(s) &= \begin{bmatrix} \mathbf{G}_{cl1}(s) \\ \mathbf{G}_{cl2}(s) \end{bmatrix} = \begin{bmatrix} \frac{u(s)}{r_F(s)} \\ \frac{y(s)}{r_F(s)} \end{bmatrix} \\ &= \begin{bmatrix} C(s)[1 + P(s)C(s)] \\ P(s)C(s)[1 + P(s)C(s)]^{-1} \end{bmatrix} \end{aligned} \quad (2)$$

We assume that the sampled frequency response

Received 2013-04-12, accepted 2013-11-05.

^{*} Supported by the National Natural Science Foundation of China (51205133), Natural Science Foundation of Shanghai (11ZR1409000), Ph.D. Programs Foundation of Ministry of Education of China (20110074120007).

^{**} To whom correspondence should be addressed. E-mail: agile_hu@ecust.edu.cn

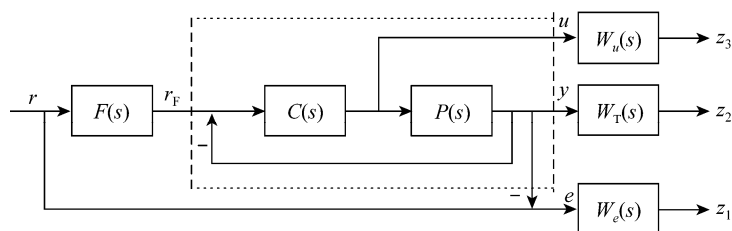


Figure 1 Control scheme for a chemical process

$\{G_{cl}(j\omega_k)\}$, $\omega_k = \frac{\pi k}{N}$, $k = 0, \dots, N$ is available ei-

ther by taking spectrum measurements or by computation of Eq. (2). The control signal u may or may not be conveniently accessible, dependent on the system hardware implementation. We try to find $F(s)$ with discrete-time frequency domain as follows

$$\left| \frac{z_1}{r} \right| = \left| \frac{e}{r} W_e(s) \right| = \left| W_e(s) \frac{1 + G(s) - F(s)G(s)}{1 + G(s)} \right| \approx 0 \quad (3)$$

$$\left| \frac{z_2}{r} \right| = \left| \frac{y}{r} W_T(s) \right| = |W_T(s) G_{cl2,1}(s) F(s)| \lesssim 1 \quad (4)$$

$$\left| \frac{z_3}{r} \right| = \left| \frac{u}{r} W_u(s) \right| = |W_u(s) G_{cl1,1}(s) F(s)| \lesssim 1 \quad (5)$$

where the sign “ $\approx$ ” means “close to” and “ $\lesssim$ ” means close to but not larger than. Eqs. (4) and (5) are specified for model-matching and control-bounding, respectively. Design for weighting functions $W_e(s)$, $W_u(s)$ and $W_T(s)$ is described as follows.

Since the constraint Eqs. (3)–(5) can be satisfied by limiting the following singular value at each frequency, it is possible to approximately transform time domain specifications into the frequency domain by suitable selections of $W_T(s)$ and $W_u(s)$.

$$\bar{\sigma} \left(\begin{bmatrix} \left| \frac{z_1}{r} \right| & \left| \frac{z_2}{r} \right| & \left| \frac{z_3}{r} \right| \end{bmatrix} \right) = \left(\frac{e}{r} W_e(j\omega) \right)^2 + \left(\frac{y}{r} W_T(j\omega) \right)^2 + \left(\frac{u}{r} W_u(j\omega) \right)^2 \leq \gamma, \quad \forall \omega \quad (6)$$

For example, to bound the control effort using a first order low pass filter

$$U_{\text{bnd}}(s) = \frac{1}{W_u(s)} = \frac{a}{1 + s/\omega_u} \quad (7)$$

where a determines the maximum allowable control effort in the frequency domain. When the input is a sine-wave, a is actually the maximum amplitude of the control signal. We assume that a can be adjusted as a reasonable parameter for time domain constraints, especially when the input signal is not known exactly. In some cases, the initial spikes of $u(k)$ can be reduced by reducing ω_u . Similarly a standard two order model is described by Franklin *et al* [18].

$$\frac{1}{W_T(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad (8)$$

We define $t_s = \frac{4.6}{\zeta\omega_n}$ (1% settling time) and $t_r \cong \frac{1.8}{\omega_n}$

(rise time for $\zeta \cong 0.5$). $W_T(s)$ and $W_u(s)$ are usually iteratively designed and verified by simulation until the response of $y(k)$ is satisfactory, with reasonable $u(k)$. This scheme is similar to the design of frequency-varying weighting filters for H_2 or H_∞ control [19, 20].

We start with the block diagram in Fig. 1. At selected frequencies, using standard convex optimization techniques, we can solve a sampled H_∞ optimization problem as follows

$$\begin{aligned} \min_{F(j\omega_k)} \max \left| \frac{z_1}{r} \right| &= \min_{F(j\omega_k)} \max \left| \frac{e}{r} W_e(j\omega_k) \right| \\ &= \min_{F(j\omega_k)} \max \left| W_e(j\omega_k) \frac{1 + G(j\omega_k) - F(j\omega_k)G(j\omega_k)}{1 + G(j\omega_k)} \right| \end{aligned} \quad (9)$$

s.t.

$$\begin{cases} \left| \frac{z_2}{r} \right| = |W_T(j\omega_k) G_{cl2,1}(j\omega_k) F(j\omega_k)| \leq 1 \\ \left| \frac{z_3}{r} \right| = |W_u(j\omega_k) G_{cl1,1}(j\omega_k) F(j\omega_k)| \leq 1, \quad \forall \omega_k; \\ k = 0, \dots, N \end{cases}$$

It means that at each selected frequency, $F(j\omega_k)$ is trying to match the reference input signal as close as possible and bounds the worst-case energy gain from the reference input to the control output. Problem (9) can be transformed to a set of linear matrix inequalities in terms of the real and complex parts of $F(j\omega_k)$. It can be solved by a constrained quadratic programming optimizer. The periodic causality constraint is described and applied [21, 22].

2.2 Design of optimal control

In this section, the complete set is obtained in parameter space (x_1, x_2) by analyzing the frequency response data of the closed loop. These designs can be formulated as convex problems and solved efficiently

by interior point methods [23].

Let controller $F(s)$ be a one order low-pass filter,

$$F(s) = \frac{x_1}{s + x_2} \quad (10)$$

We make the following assumptions throughout the paper.

(1) The loop is stabilizable, that is, polynomials $N(s)$ and $D(s)$ are coprime.

(2) The loop has no $j\omega$ poles.

(3) The only information available to the designer is: ① the knowledge of frequency response magnitude and phase, *i.e.*, $P(j\omega) = P_r(\omega) + jP_i(\omega)$, for $\omega \in [0, \infty]$; and ② the knowledge of the number of plant right half-plane (RHP) poles.

The root invariant regions in parameter space (x_1, x_2) can be determined using the following result. Consider the characteristic polynomial of the two-degree-of-freedom control system:

$$\Pi(s) = x_1 N(s) + (s + x_2) D(s) \quad (11)$$

Polynomials $N(j\omega)$ and $D(j\omega)$ are defined as

$$\begin{cases} N(j\omega) = N_r(\omega) + jN_i(\omega) \\ D(j\omega) = D_r(\omega) + jD_i(\omega) \end{cases} \quad (12)$$

And

$$\begin{aligned} G(j\omega) &= \frac{N(j\omega)}{D(j\omega)} \\ &= \frac{N_r(\omega)D_r(\omega) + \omega^2 N_i(\omega)D_i(\omega)}{D_r^2(\omega) + \omega^2 D_i^2(\omega)} + \\ &\quad j \frac{\omega[N_i(\omega)D_r(\omega) - N_r(\omega)D_i(\omega)]}{D_r^2(\omega) + \omega^2 D_i^2(\omega)} \\ &= G_r(\omega) + jG_i(\omega) \end{aligned} \quad (13)$$

Hence, Eq. (11) can be written as

$$\begin{aligned} \Pi(j\omega) &= x_1 N(j\omega) + (j\omega + x_2) D(j\omega) \\ &= x_1 [N_r(\omega) + jN_i(\omega)] + \\ &\quad (j\omega + x_2) [D_r(\omega) + jD_i(\omega)] \\ &= R(\omega) + jI(\omega) \end{aligned} \quad (14)$$

where

$$R(\omega) = x_1 N_r(\omega) + x_2 D_r(\omega) - \omega^2 D_i(\omega) \quad (15)$$

$$I(\omega) = x_1 N_i(\omega) + x_2 D_i(\omega) + D_r(\omega) \quad (16)$$

Based on the boundary crossing theorem [24], we have following two conditions.

(1) Real root crossing condition

From Eq. (11), it follows

$$\Pi(0) = x_1 N(0) + x_2 D(0) = 0 \quad (17)$$

and since $D(0) \neq 0$, it is equivalent to

$$x_1 \frac{N(0)}{D(0)} + x_2 = x_1 G(0) + x_2 = 0 \quad (18)$$

(2) Complex root crossing condition

By setting Eqs. (15) and (16) to be zero, we have

$$x_1(\omega) = -\frac{D_r^2(\omega) + \omega^2 D_i^2(\omega)}{N_i(\omega)D_r(\omega) - N_r(\omega)D_i(\omega)} = -\frac{\omega}{G_i(\omega)} \quad (19)$$

$$x_2(\omega) = -\frac{N_r(\omega)D_r(\omega) + \omega^2 N_i(\omega)D_i(\omega)}{N_i(\omega)D_r(\omega) - N_r(\omega)D_i(\omega)} = \frac{G_r(\omega)\omega}{G_i(\omega)} \quad (20)$$

One straight line and one curve can be deduced in the plane x_1 - x_2 based on the frequency domain data $G(j\omega)$ of the loop. Once the controller parameter space is partitioned into root invariant regions, it is necessary to select a point from each region and test the stability of the corresponding closed-loop system. The stability test can easily be done by plotting the Nyquist plot and from the knowledge of the number of RHP poles of the loop.

2.3 Stability analysis

In control system design, additional requirements beyond closed-loop stability are imposed. These additional requirements include guaranteed gain and phase margin, and acceptable bounds on the H_∞ norm of the closed loop transfer function. We propose a new robustness specification λ , defined as

$$\lambda = \max_{0 \leq \omega < \infty} |\operatorname{Re}[G(j\omega)]| \quad (21)$$

where $G(j\omega)$ is the closed loop transfer function and λ is the maximum of absolute real part of $G(j\omega)$, as shown in Fig. 2. The new specification is similar to the gain margin and phase margin specifications.

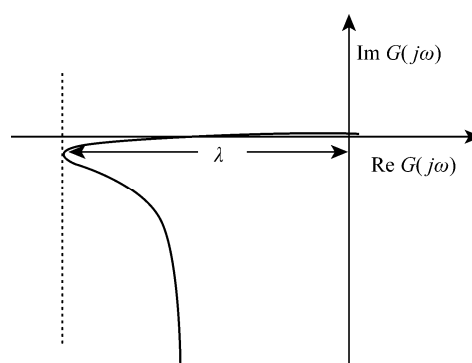


Figure 2 Geometrical illustration of constraint (21)

Such a problem is equivalent to the simultaneous stabilization problem of the original closed loop $G(s)$ and the family loop transfer function $G'(s)$ whose frequency responses are defined as follows.

Guaranteed gain margin problem

$$G'(j\omega_k) := \{kG(j\omega_k) : k \in [1, k^*]\}, \quad A_m > \frac{1}{\lambda} \quad (22)$$

where $k^* \geq 1$ is the required gain margins. If conditional gain margin is required, it can be treated similarly.

Guaranteed phase margin problem

$$G'(j\omega_k) := \{e^{j\theta} G(j\omega_k) : \theta \in [0, \theta^*]\}, \varphi_m > \arccos(\lambda) \quad (23)$$

where $\theta^* \geq 0$ is the required phase margins.

Guaranteed H_∞ margin problem

For sensitivity function $S(s)$,

$$\|W(s)S(s)\|_\infty < \gamma,$$

$$G'(j\omega_k) := \left\{ G(j\omega_k) \frac{1}{1 + \frac{1}{\gamma} e^{j\theta} W(j\omega_k)} : \theta \in [0, 2\pi] \right\} \quad (24)$$

For complementary sensitivity function $T(s)$,

$$\|W(s)T(s)\|_\infty < \gamma,$$

$$G'(j\omega_k) := \left\{ G(j\omega_k) \left[1 + \frac{1}{\gamma} e^{j\theta} W(j\omega_k) \right] : \theta \in [0, 2\pi] \right\} \quad (25)$$

The family $G'(s)$ is in general a complex family.

3 SIMULATION RESULTS

In this section we will give two examples to illustrate the design method for the first order low pass feedforward controller. We collect the frequency domain (Nyquist-Bode) data of the loop transfer functions for following processes.

Example 1 Consider plant $P(s)$ and controller $C(s)$ analyzed by Piazzoli and Visioli [25] with the closed loop transfer function $G(s)$ referring to $G'(j\omega_k) := \{G(j\omega_k) = C(j\omega_k)P(j\omega_k) : \omega_k \in (0, 1000] \text{ sampled every } 0.01\}$. The Nyquist plot of $G(j\omega)$ obtained is shown in Fig. 3, in which the closed loop stabilization is satisfied for all frequencies.

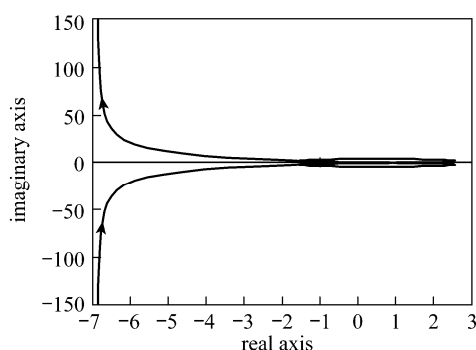


Figure 3 Nyquist plot of $G(j\omega)$ for Example 1

From the data $G(j\omega)$, we have $G(0) = 3.156$ and $G(\infty) = 0$. The data points representing curves x_1 and x_2 in Eqs. (19) and (20) are depicted in Fig. 4. By testing each point for the stabilization, it is identical to those in the example.

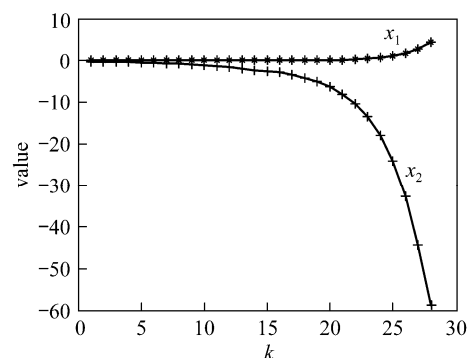


Figure 4 Curves x_1 and x_2 for Example 1

We now consider the system stabilization. The control system transfer function $H(j\omega)$ can be defined as $H(j\omega) = F(j\omega)G(j\omega)$. Fig. 5 shows the model and true control system bode plot of $H(j\omega)$ obtained.

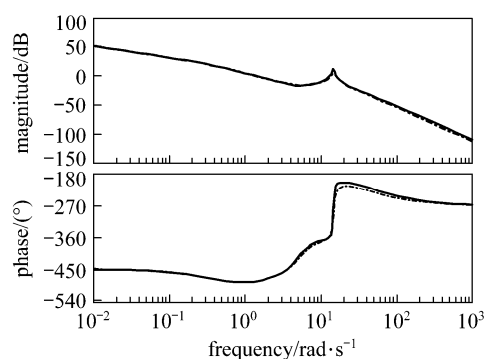


Figure 5 Bode plot of $H(j\omega)$ model (dashed) and true (solid) frequency responses for Example 1

The bode plot shown the closed loop stabilization is satisfied for all frequencies. The model frequency responses fit true data.

Example 2 Consider the process with a large time delay analyzed by Wang and Shao [26], $P(s) =$

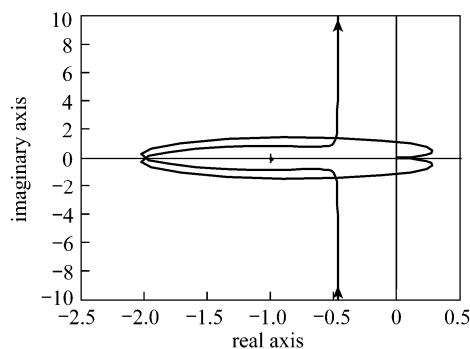
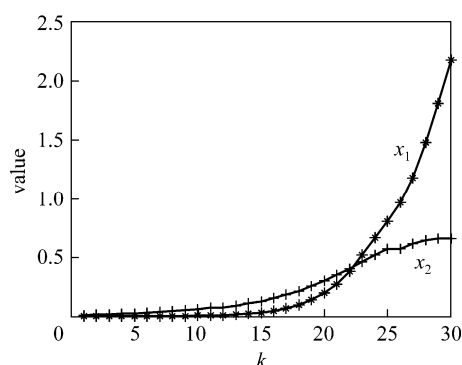
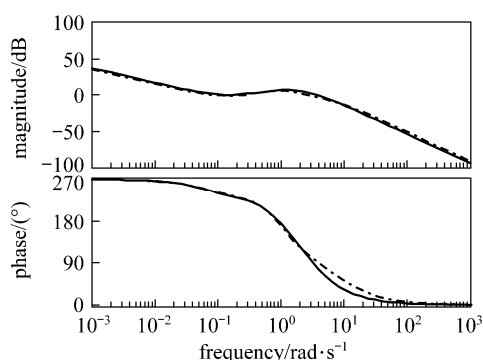
$$\frac{1}{(s+1)(s+2)(s+3)} e^{-10s}, \quad C(s) = 1.9815 + \frac{0.4032}{s},$$

and the closed loop transfer function $G(s)$ refers to $G'(j\omega_k) := \{G(j\omega_k) = C(j\omega_k)P(j\omega_k) : \omega_k \in (0, 1000] \text{ sampled every } 0.01\}$.

The Nyquist plot of $G(j\omega)$ is shown in Fig. 6. The closed loop stabilization is satisfied for all frequencies. From the data $G(j\omega)$, we have $G(0) = 0.012$ and $G(\infty) = 0$. The data points representing curves of x_1 and x_2 in Eqs. (19) and (20) are depicted in Fig. 7.

For the system stabilization, the model and true control system bode plot of $H(j\omega)$ are shown in Fig. 8.

The bode plot shows that the closed loop stabilization is satisfied for all frequencies. At higher frequencies, the model frequency response causes a little error compared with true data because the feedforward controller selected is a low pass filter.

Figure 6 Nyquist plot of $G(j\omega)$ for Example 2Figure 7 Curve of x_1 and x_2 for Example 2Figure 8 Bode plot of $H(j\omega)$ model (dashed) and true (solid) frequency responses for Example 2

Using standard convex optimization techniques and minimizing the tracking errors, we can solve a sampled H_∞ optimization problem. Fig. 9 shows the mean magnitude of the tracking errors for different

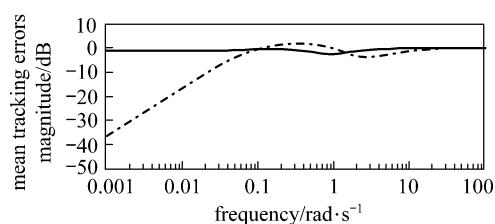


Figure 9 Mean tracking errors at different frequencies for feedback only (dashed) and feedback with feedforward control (solid)

controllers at different frequencies. The feedforward controller can improve the tracking performance.

4 CONCLUSIONS

In this paper, a new design method for a chemical system is developed. The complete set is obtained in parameter space (x_1, x_2) in terms of frequency response data of the closed loop system rather than from transfer function or state space. Satisfactory responses can be expected for process with various dynamics, including those with high order, large delay time responses. System stabilization and tracking performance are guaranteed with the frequency responses of the closed loop transfer functions. Design parameter λ satisfies the system robustness. Using standard convex optimization techniques, we can minimize the tracking errors. Simulation examples show the effectiveness and flexibility of the controller in handling processes of different characteristics. The control system can be designed without model or identification.

NOMENCLATURE

$C(s)$	feedback controller
$D(j\omega)$	frequency domain polynomial
$D(s)$	s domain polynomial
e	control error
$F(s)$	feedforward controller
$G(j\omega)$	frequency domain transfer function
$G(s)$	control system transfer function
$G_{cl}(j\omega_k)$	transfer function frequency response
$G_{cl}(s)$	transfer function matrix
$N(j\omega)$	frequency domain polynomial
$N(s)$	s domain polynomial
$P(s)$	chemical process
r	set point
S	s domain function
$S(s)$	sensitivity function
$T(s)$	complementary sensitivity function
u	control effort
$W_e(s)$	frequency-varying weighting functions for the control error e
$W_T(s)$	frequency-varying weighting functions for the process output y
$W_u(s)$	frequency-varying weighting functions for the control effort u
x_1	parameter space
x_2	parameter space
y	process output
ζ	coefficient
θ	phase
λ	robustness specification
$\Pi(s)$	characteristic polynomial of the control system
$\Pi(j\omega)$	frequency domain characteristic polynomial of the control system
ω_k	sample width

Subscripts

e	control error
i	imaginary part
k	number
r	real part
T	process output
u	control effort

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